CHILE–HONDURAS MEETING

Differential Equations in Biological Sciences and Engineering

University of Talca

Faculty of Agricultural Sciences

BOOK OF ABSTRACTS

07 August 2026

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University of Talca
Food Production Research Center (NIPA),
Catholic University of Temuco

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Introduction

The history behind this meeting is deeply rooted in academic collaboration, friendship, and a shared passion for mathematics across Latin America.

The origins of this initiative can be traced back to Dr. Marlon López-Flores, a Honduran mathematician whose academic career led him from Honduras to advanced studies in Brazil. In 2010, during a Summer School at the Instituto de Matemática Pura e Aplicada (IMPA), Dr. López-Flores met the Chilean mathematician Dr. William Campillay-Llanos. During that same event, they also met a young mathematics enthusiast, José Ramón Madrid Padilla, who would later become an internationally recognized researcher. Although their careers would develop along different paths, the scientific ties established during those formative years remained active.

Fifteen years later, in 2025, these researchers met again during the 35th Brazilian Mathematics Colloquium. Motivated by common interests in mathematical modeling, differential equations, dynamical systems, and interdisciplinary applications, they began discussing the possibility of establishing a long-term scientific collaboration between Chile and Honduras. These conversations eventually led to the creation of a broader academic initiative involving researchers, graduate students, and institutions from both countries.

A decisive step in this process was the formation of the **Grupo de Biomodelamiento Matemático e Inteligencia Artificial (GBM–IA)**, a collaborative research group composed primarily of young Honduran researchers from the Master's Program in Mathematics at the National Autonomous University of Honduras (UNAH), together with international collaborators. The group was conceived with the goal of developing advanced mathematical methodologies for problems arising in biological sciences, engineering, artificial intelligence, and data-driven modeling.

The scientific foundations of the GBM–IA are supported by an active research agenda in applied analysis, differential equations, numerical methods, and mathematical modeling. This collaboration has already produced several ongoing manuscripts that develop new mathematical frameworks for positive-state systems, proportional arithmetic, logarithmic discretization, and computational modeling.

Among the theoretical contributions are the manuscripts *A Proportional-Arithmetic Framework for Fourier Analysis on the Positive Real Line*, *A Positive-State Laplace Calculus for Dirichlet Resolvents and Log-Ratio Stability*, and *Logarithmic-Scale Finite Difference Schemes for Positive Parabolic Problems on Geometric Grids*. These studies establish novel analytical and numerical methodologies for the treatment of positive dynamical systems and provide the mathematical foundations for future applications in biological sciences, engineering, and artificial intelligence.

Building upon these theoretical developments, the collaboration has expanded toward data-driven mathematical modeling through the conference contribution *Proportional Discrete Fourier Analysis of WaPOR Actual Evapotranspiration over Cropland in Comayagua, Honduras*, together with the ongoing studies *A Proportional Positive-State Model for Post-Irrigation Soil-Moisture Recovery Using Open Smart-Irrigation Data* and *Multiyear Validation and Mass-Balance Auditing of Richards Simulations on Uniform and Graded Finite-Volume Meshes Using USDA SCAN Observations*. These projects combine mathematical modeling, numerical analysis, remote sensing, artificial intelligence, and environmental data to address contemporary problems in agriculture and water-resource management.

Together, these works illustrate the scientific vision of the GBM–IA: developing rigorous mathematical theory while simultaneously creating computational methodologies capable of addressing real-world challenges in biological sciences, engineering, agriculture, and artificial intelligence.

The initiative was significantly strengthened through the scientific leadership of Dr. Samuel Ortega-Farías, Director of the Research and Transfer Center for Irrigation and Agroclimatology (CITRA) at the University of Talca. Over the past decade, Dr. Ortega-Farías has been one of the principal promoters of biomathematical modeling in agricultural sciences, fostering the integration of mathematical modeling, agroclimatology, crop physiology, environmental physics, and data-driven methodologies. His interdisciplinary vision has been reflected not only in his scientific contributions but also in the development of postgraduate education, where he has actively promoted advanced training in mathematical modeling and its applications to agriculture and environmental sciences.

This vision also inspired the collaboration between Dr. Marlon López-Flores and Dr. William Campillay-Llanos at CITRA through the international academic project *Projection of Sustainable Fruit Production through Biomathematical Modeling and Artificial Intelligence* (International Academic Events Fund, 2023). This project strengthened scientific cooperation between Chile and Honduras, bringing together expertise in mathematical modeling, artificial intelligence, agroclimatology, and agricultural sciences, ultimately laying the foundations for the present international meeting.

More recently, the integration of advanced mathematical modeling, stochastic processes, and agricultural phenology has opened new interdisciplinary research directions. A representative example is the work

> Rosales, A., Ortega-Farías, S., de la Fuente-Sáiz, D., Morales-Zárate, R., Campillay-Llanos, W., Suarez, E., & Lisperguer, M. J. (2026). *Development of Predictive Models for Simulating Vegetative and Reproductive Phenology in European Hazelnut Trees (Corylus avellana L.).* International Journal of Biometeorology, 70(6), 184.

which illustrates how mathematical and stochastic modeling can improve the understanding of plant development under changing environmental conditions, providing robust tools for precision agriculture, climate adaptation, and sustainable crop management.

To further consolidate this collaborative effort, Dr. Braulio Soto, Director of the Núcleo de Investigación en Producción Alimentaria (NIPA) and Editor-in-Chief of the *Journal of Natural Resources and Farming*, joined the scientific and organizing committee. His participation strengthened the institutional framework of the initiative and promoted the creation of the Special Issue entitled *Mathematical and Computational Modelling of Agricultural and Natural Resource Systems*, providing an international publication venue aligned with the scientific objectives of the meeting.

This collaboration culminated in the organization of the

Chile–Honduras Meeting

Differential Equations in Biological Sciences and Engineering

María Clara Fittipaldi

Title

Asymptotic behavior of a logistic epidemiological model with random transmission and treatment rates

Affiliation:
Faculty of Sciences, National Autonomous University of Mexico (UNAM), Mexico

Abstract

In this talk, we present an extension of the stochastic logistic treatment model for AIDS transmission and control introduced by Ding, Xu, and Hu (2009), itself a modification of the classical Roberts and Saha (1999) model, by incorporating a second independent Brownian noise source.

Unlike the Ding-Xu-Hu model, whose dynamics are perturbed by a single environmental noise source, our formulation includes two distinct stochastic perturbations acting simultaneously on the proportion of infected individuals. This leads to a stochastic differential equation with logistic drift and a diffusion term whose intensity depends on the infected proportion.

We show that the model admits an equivalent-in-law representation driven by a single Brownian motion constructed through an adapted combination of the original noises. Using this equivalent representation, we derive a stochastic reproduction number separating extinction and persistence regimes, and compare our persistence threshold with those obtained in the Roberts-Saha and Ding-Xu-Hu models.

This is joint work with Alejandra Christen (University of Valparaíso, Chile) and José Miguel Zúñiga Núñez (Catholic University of Maule, Chile).

Keywords:
Stochastic differential equations, epidemiological models, persistence, extinction, AIDS dynamics.

Ángela Carolina Tunubalá Sánchez

Title

Limit Cycles on Hybrid Systems

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Abstract

A hybrid dynamical system is one whose behavior is governed by both continuous and discrete dynamics; that is, it exhibits both flows and jumps. The field of hybrid dynamical systems is relatively recent and encompasses a broad range of phenomena and is often used to model natural processes. In this work, we study the maximum number of limit cycles in some classes of discontinuous piecewise hybrid differential systems formed by two rigid systems separated by a straight line. These rigid systems are formed by a linear center plus a homogeneous polynomial of degree 2, 3, 4, 5, and 6. Interest in studying limit cycles of discontinuous piecewise hybrid differential systems has increased in recent years due to their wide range of applications and their connection to the classical *Hilbert's 16th problem*, which asks whether there exists an upper bound for the number of limit cycles in planar polynomial vector fields of a given degree. At present, this problem remains open. Our main objective is to determine the maximum number of limit cycles that can arise in these systems under study; that is, to solve the extended version of *Hilbert's 16th problem* for this class of discontinuous piecewise differential systems. The main result states that the maximum number of limit cycles for this class of systems is at most one when $m = n$, and this bound is achieved. For $m = 2$ and $n = 3, 4, 5, 6$, the maxima are $2, 3, 4$, and 5, respectively. For $m = 3$ and $n = 4, 5, 6$, the corresponding bounds are $4, 2$, and 2. In the case $m = 4$, the maximum is 6 for $n = 5$ and 10 for $n = 6$. Finally, when $m = 5$ and $n = 6$, the system admits at most 8 limit cycles. In this way, we solve the extended version of *Hilbert's 16th problem* for a class of discontinuous piecewise hybrid differential systems separated by the straight line $y = 0$ and formed by two rigid polynomial differential systems. This is joint work with Jaume Llibre (Autonomous University of Barcelona, Spain) and Durval J. Tonon (Federal University of Goiás, Brazil).

Keywords:
Hybrid systems, limit cycles, Poincaré map.

José Ramón Madrid Padilla

Title

What Is the Smoothest Local Average?

Affiliation:
Virginia Tech, USA

Abstract

Averaging is one of the simplest ways to smooth discrete data, but different averaging kernels can produce very different levels of regularity. This leads to a natural extremal question: among all local averages supported at a fixed scale, which one minimizes the largest possible higher-order oscillation of the resulting signal?

We formulate this problem using convolution operators on the integers and measure smoothness through higher-order finite differences. Fourier analysis transforms the operator-norm minimization problem into an extremal problem for algebraic polynomials on an interval. For first- and second-order differences, the optimal kernels are the familiar uniform and triangular averages. For higher-order differences, however, new phenomena appear: sharp solutions are governed by Chebyshev and Zolotarev polynomials, and in some cases the optimal constants involve elliptic functions.

The talk will describe this connection between optimal discrete smoothing, Fourier multipliers, and classical extremal polynomial theory.

This is joint work with José Gaitán and Carlos Garzón.

Keywords:
Discrete smoothing; convolution operators; finite differences; Fourier analysis; extremal polynomials; Chebyshev polynomials; Zolotarev polynomials.

Emilio Vilches

Title

Plenary Lecture 4

Affiliation:
Universidad de O'Higgins (UOH), Chile

Abstract

Abstract to be provided.

Keywords:
keywords to be provided.

Rosibel Pacheco

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Presidenta de la Sociedad Matemática de Honduras (ASOMATH), Honduras

Abstract

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Keywords:
keywords to be provided.

William Campillay-Llanos

Title

From Classical Phenological Models to Reaction–Diffusion Equations: A Spatial Bioclimatic Framework for Cherry Phenology

Authors:
William Campillay-Llanos

Affiliation:
Núcleo de Investigación en Producción Alimentaria (NIPA), Facultad de Recursos Naturales, Universidad Católica de Temuco, Chile

Abstract

Classical phenological models based on growing degree days have been widely used to predict sweet cherry development. However, these approaches are generally formulated at individual locations, neglecting the spatial variability generated by topography, microclimate, soil heterogeneity, and orchard management.

This work proposes a mathematical framework that extends classical phenological models from ordinary differential equations to reaction–diffusion partial differential equations by interpreting phenology as a continuous spatio-temporal field. The reaction term represents local phenological development driven by thermal accumulation, while the diffusion term captures the effective spatial continuity of environmental conditions and phenological progression throughout the orchard.

The model incorporates spatially distributed agroclimatic information and is solved numerically using finite-difference methods implemented in `MATLAB`, allowing the simulation of current and future climate scenarios. Compared with conventional point-based or interpolation approaches, the proposed framework provides a mechanistic description that simultaneously integrates spatial and temporal dynamics while preserving the biological interpretation of phenological development.

This work establishes a mathematical basis for digital phenology and precision horticulture and provides a flexible framework for future coupling with epidemiological, hydrological, and crop growth models. The study is the result of a collaborative effort combining mathematical modelling and agroclimatology, carried out jointly with Samuel Ortega-Farías and Marlon M. López-Flores.

Keywords:
Reaction–diffusion equations; Cherry phenology; Mathematical modelling; Digital phenology; Partial differential equations; Precision agriculture; Climate change.

Rafael López-Olivarí

Title

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Affiliation:
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Abstract

Abstract to be provided.

Keywords:
keywords to be provided.

Emilio Jorquera Fontena

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Abstract

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Keywords:
keywords to be provided.

Hailen Millailo Quintana

Title

Title to be provided.

Affiliation:
INACAP, Chile

Abstract

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Keywords:
keywords to be provided.

Gustavo Maldonado

Title

A Proportional Positive-State Model for Post-Irrigation Soil-Moisture Recovery Using Open Smart-Irrigation Data

Affiliation:
Universidad Nacional Autónoma de Honduras (UNAH), Honduras

Abstract

Soil moisture is a positive state variable that is commonly interpreted relative to an agronomic or empirical reference level. This work presents a proportional positive-state model for describing post-irrigation soil-moisture recovery by directly modeling the normalized moisture ratio rather than additive moisture differences. The proposed formulation is based on proportional calculus, where the equilibrium state corresponds to the multiplicative reference value, yielding a first-order proportional relaxation model with an explicit analytical solution.

The methodology was evaluated using an open smart-irrigation dataset collected from a commercial strawberry farm in Paraguay, including soil-moisture measurements, flow-meter records, and electrovalve status. Irrigation events were automatically identified from flow-meter data, recovery periods were extracted, and the proportional model was compared with a classical exponential relaxation model through logarithmic error metrics, chronological hold-out validation, bootstrap analysis, and sensitivity tests.

Results showed that the proportional formulation consistently reduced logarithmic prediction errors across most irrigation events while preserving a simple and interpretable mathematical structure. The proposed model provides a compact event-level description of soil-moisture recovery and offers an alternative framework for modeling positive-state variables when the scientific objective is to describe relative recovery toward a reference condition.

Keywords:
Proportional calculus; Soil moisture; Smart irrigation; Positive-state model; Mathematical modelling; Open data.

José David Zúñiga

Title

Multiyear Validation and Mass-Balance Auditing of Richards Simulations on Uniform and Graded Finite-Volume Meshes Using USDA SCAN Observations

Affiliation:
Universidad Nacional Autónoma de Honduras (UNAH), Honduras

Abstract

The Richards equation is one of the most widely used mathematical models for simulating water movement in variably saturated soils. However, numerical accuracy depends strongly on spatial discretization, temporal integration, and the preservation of water mass balance. This work presents a multiyear validation framework for one-dimensional finite-volume simulations using publicly available USDA SCAN soil-moisture observations.

The methodology combines calibration on a single growing season with independent temporal validation over two additional seasons, allowing numerical discretization errors to be distinguished from structural model uncertainty. Uniform, logarithmic, and algebraic graded meshes are compared through convergence analysis, while an independent mass-balance audit evaluates the consistency of temporal integration under refined predictor–corrector schemes.

Results show that surface-focused graded meshes consistently reduce spatial discretization errors and that this improvement transfers across independent validation periods. Nevertheless, numerical mesh errors remain substantially smaller than model–observation discrepancies, indicating that predictive uncertainty is primarily governed by environmental forcing, model structure, and observational uncertainty rather than grid resolution alone. The proposed framework provides a transparent benchmark for multiyear validation, mesh selection, and water-balance auditing in Richards equation simulations, supporting the development of reproducible computational tools for soil-water modelling.

Keywords:
Richards equation; Finite-volume method; Soil moisture; Numerical validation; Graded mesh; Mass-balance audit; Mathematical modelling.

Carlos Miguel Cruz

Title

Proportional Discrete Fourier Analysis of WaPOR Actual Evapotranspiration over Cropland in Comayagua, Honduras

Affiliation:
Universidad Nacional Autónoma de Honduras (UNAH), Honduras

Abstract

Actual evapotranspiration is a strictly positive environmental variable whose interpretation is frequently based on ratios, percentage changes, and seasonal contrasts. This work presents a proportional discrete Fourier framework for analyzing positive agronomic time series by combining proportional arithmetic with the classical discrete Fourier transform. The proposed methodology interprets the logarithmic representation of the series as a proportional space, allowing seasonal variability to be analyzed relative to the geometric mean rather than through absolute deviations.

The methodology was applied to monthly WaPOR actual evapotranspiration (AETI) data extracted from cropland areas in Comayagua, Honduras, using publicly available datasets from FAO WaPOR, FAO GAUL, and ESA WorldCover. The proportional spectrum was compared with the conventional Fourier spectrum to identify dominant seasonal components and evaluate relative versus absolute variability.

Results showed that both approaches identified a dominant annual cycle, while the proportional formulation provided a unit-invariant interpretation of seasonal variability that naturally emphasizes relative changes around the geometric baseline. Rather than replacing classical Fourier analysis, the proposed framework extends its interpretation for strictly positive environmental variables, offering a useful mathematical tool for remote sensing, digital agriculture, and environmental monitoring.

Keywords:
Proportional arithmetic; Discrete Fourier transform; WaPOR; Evapotranspiration; Remote sensing; Mathematical modelling; Precision agriculture.

Marlon M. López-Flores

Title

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Affiliation:
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Abstract

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Keywords:
keywords to be provided.

Samuel Ortega-Farías

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Abstract

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Keywords:
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Keywords:
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Meszéna Géza

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Affiliation:
International Initiative for Theoretical Ecology, Hungary

Abstract

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Keywords:
keywords to be provided.

Braulio Soto

Title

Presentation of the Chile–Honduras Special Issue: *Mathematical and Computational Modelling of Agricultural and Natural Resource Systems*

Affiliation:
Editor-in-Chief, Journal of Natural Resources and Farming (JONRAF)
Universidad Católica de Temuco (UCT), Chile

Abstract

The *Journal of Natural Resources and Farming (JONRAF)* is an international, peer-reviewed, Diamond Open Access journal published by the Universidad Católica de Temuco. The journal disseminates high-quality research in agricultural sciences, natural resources, environmental sciences, forestry, ecology, and related interdisciplinary fields, promoting scientific contributions that support sustainable development and innovation.

As part of the *Chile–Honduras Meeting on Differential Equations in Biological Sciences and Engineering*, JONRAF is pleased to present the Special Issue entitled *Mathematical and Computational Modelling of Agricultural and Natural Resource Systems*. This Special Issue aims to bring together original research that advances mathematical, computational, and data-driven approaches to biological, agricultural, and environmental systems. Contributions involving differential equations, numerical methods, artificial intelligence, optimization, stochastic modeling, remote sensing, and digital agriculture are particularly encouraged.

Mathematical modeling plays an increasingly important role in modern science by providing complementary perspectives to experimental and observational research. Mathematical models make it possible to formulate hypotheses, identify underlying mechanisms, quantify uncertainty, predict system behavior under alternative scenarios, and integrate knowledge across disciplines. As biological and agricultural systems become increasingly complex, mathematical and computational approaches offer rigorous tools that complement empirical studies and support evidence-based decision-making in research, technology, and sustainable resource management.

For additional information about the journal, its editorial scope, and submission guidelines, please visit the official website:

`https://jonraf.uct.cl/index.php/jonraf/en`

Keywords:
Special Issue; Journal of Natural Resources and Farming; mathematical modeling; computational modeling; differential equations; agricultural sciences; natural resources; artificial intelligence; sustainable agriculture.

Epilogue

The *International Meeting on Differential Equations in Biological Sciences and Engineering* represents far more than a scientific event. It is the beginning of a long-term initiative aimed at building an international community where mathematics serves as a common language for understanding and solving complex real-world problems.

This first edition has brought together researchers, students, and institutions from Chile, Honduras, Brazil, Mexico, Spain, Hungary, and other countries, creating new opportunities for collaboration across diverse areas of mathematics and its applications. By connecting expertise in differential equations, mathematical modeling, numerical analysis, stochastic processes, artificial intelligence, computational mathematics, and interdisciplinary applications in biology, agriculture, environmental sciences, and engineering, the meeting seeks to strengthen scientific cooperation while inspiring new generations of researchers.

Beyond the lectures and discussions, the true legacy of this meeting lies in the collaborations that will emerge from it: joint research projects, student exchanges, scientific publications, graduate supervision, and international networks capable of addressing today's global challenges through rigorous mathematical thinking. We hope that the ideas shared during these sessions will continue to evolve into long-lasting partnerships and innovative research with meaningful scientific and societal impact.

The organizing committee sincerely thanks all invited speakers, participants, collaborating institutions, scientific societies, reviewers, and sponsors whose enthusiasm and commitment made this inaugural edition possible. Their support has laid the foundation for what we hope will become a distinguished international forum for mathematical sciences.

Finally, we warmly invite the mathematical community to remain connected and to stay tuned for the next edition of this meeting. We look forward to welcoming both returning participants and new colleagues as we continue expanding this international network, fostering interdisciplinary collaboration, and advancing the role of mathematical modeling in science, engineering, and society.